Log-Log Graphs

When we graph data in physics, the goal is to find the mathematical relationship between the things we are graphing. Typically, this means finding the slope of a best-fit line. Occasionally, the data we are graphing is not linear, and in those cases we "linearize" the data until we can make a straight line. Linearizing is usually straight forward because most of introductory physics are relatively straight forward equations. But what if you are having trouble linearizing the data? What if you really have no idea what the power relationship between the data is? Thankfully, there is another way to determine the mathematical relationship: graphing the logarithms of the data!

To see how this is helpful, let's just do some math first. Imagine we have some power relationship between y and x given by

$$y = ax^n$$

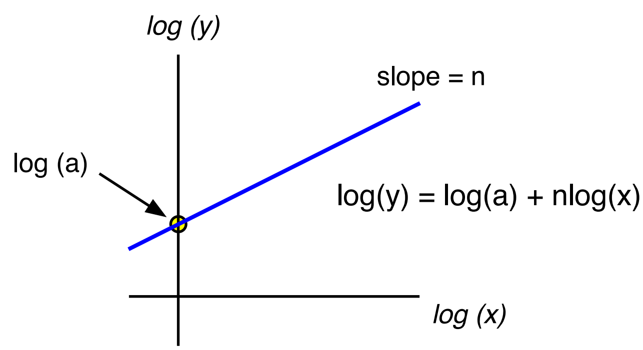
In the equation above, n is the power of the relationship and a is the coefficient of the relationship. Taking the logarithm of both sides gives

$$\log(y) = \log(ax^n)$$

Which equals

$$\log(y) = \log(a) + n\log(x)$$

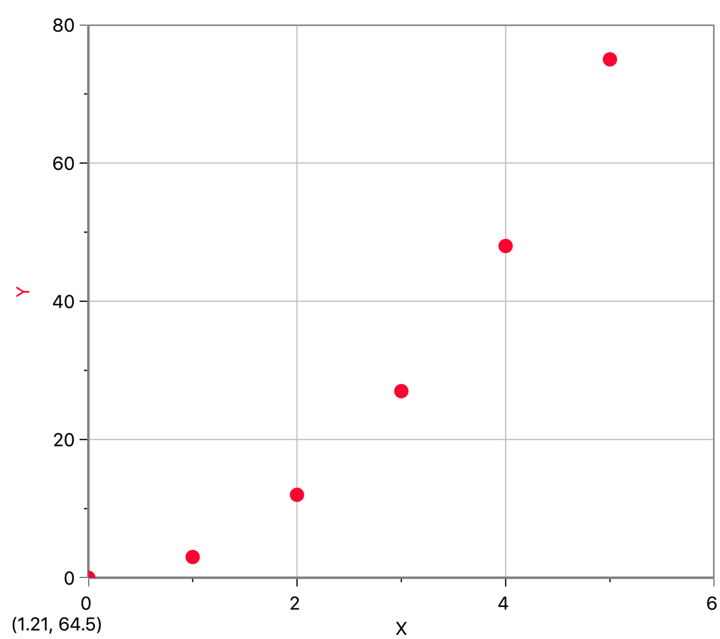
Graphing log (y) vs log (x) would thus give the following graph:



This is extremely useful as the slope of the graph tells us the power relationship between the two variables and the intercept can tell us the coefficient in the relationship. This will make more sense with an example.

Let's say we are trying to find the relationship between y and x for the following data with traditional graph:

x	y
0	0
1	3
2	12
3	27
4	48
5	75



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If you like finding math patterns, you probably already figured out the relationship, but let's assume you don't know. Now look at what happens when we graph the *logarithms* of the data:

Log (x)	*Log (y)*
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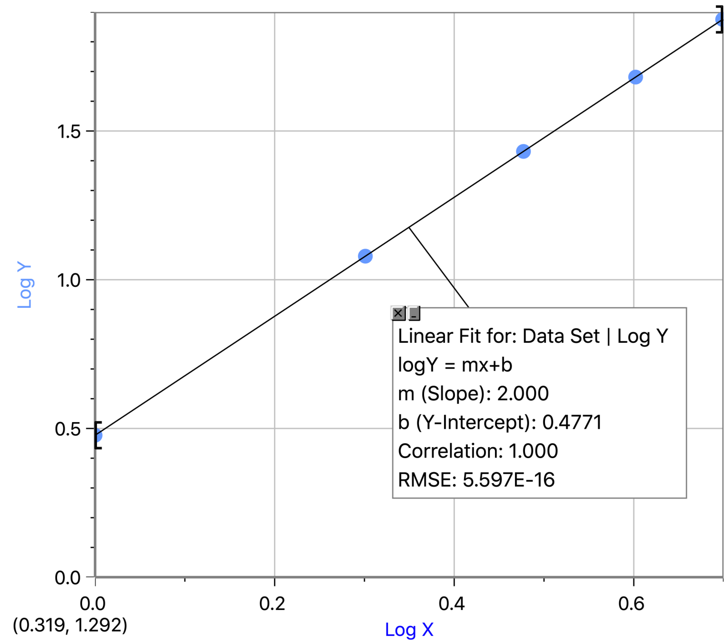
0.000	0.477
0.301	1.079
0.477	1.431
0.602	1.681
0.699	1.875



We end up with a straight line! From the Logger Pro results, our equation would be

$$\log\ (y) = 0.477 + 2\log\ (x)$$

Since the slope of the line is 2, the exponent on x is 2. The vertical intercept of 0.4771 is the logarithm of the x coefficient:

$$0.477 = \log\ (a)$$

which means

$$a = 10^{0.4771} = 3$$

Putting this all together, we can say that the equation that matches our data is

$$y = 3x^2$$

Please notice that it doesn't matter what base we use for the logarithms. If we had done natural logarithms instead, the best fit straight line would have been

$$\ln\ (y) = 1.099 + 2\ln\ (x)$$

The power is still "2" and then to calculate the coefficient we say

$$1.099 = \ln\ (a)$$

which means

$$a = e^{1.099} = 3$$

We end up with the same result.

"Logarithms are great!"